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UV/IR mixing in noncommutative QED defined by Seiberg-Witten map

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Abstract: Noncommutative gauge theories defined via Seiberg-Witten map have desirable properties that theories defined directly in terms of noncommutative fields lack, covariance and unrestricted choice of gauge group and charge being among them, but nonperturbative results in the deformation parameter θ are hard to obtain. In this article we use a θ-exact approach to study UV/IR mixing in a noncommutative quantum electrodynamics (NCQED) model defined via Seiberg-Witten map. The fermion contribution of the one loop correction to the photon propagator is computed and it is found that it gives the same UV/IR mixing term as a NCQED model without Seiberg-Witten map.

Keywords: Non-Commutative Geometry, Gauge Symmetry, Nonperturbative Effects.

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Contents

1. Introduction

Noncommutative quantum electrodynamics (NCQED) is usually defined in analogy to Yang-Mills theory with matrix multiplication replaced by star products. The resulting action is invariant under noncommutative gauge transformations. Such models appear quite naturally in certain limits of string theory in the presence of a background B-field [1], they can also be used to gain some understanding about phenomenological implications of a quantum structure of spacetime. One of the particularly intriguing effects is UV/IR-mixing, an interrelation between short and long-distance scales that is absent in ordinary quantum field theory. There are however some problems with this simple definition of NCQED: (1) The possible choices of charges for particles are restricted to ± 1 or 0 times a fixed unit of charge and in the nonabelian case the choice of structure group is limited to $\mathrm{U}(N)$ in the fundamental representation. (2) An ordinary gauge field $a_\mu(x)$ transforms like a vector under a change of coordinates, $a'_\mu(x') = \partial x^\nu/\partial x'^\mu a_\nu(x)$, while for the fields $A_\mu(x)$ of NCQED this holds only for rigid, affine coordinate changes [2, 3]. The solution to both problems is an alternative approach to noncommutative gauge theory based on Seiberg-Witten maps. This approach to noncommutative gauge theory (especially, the noncommutative extension of nonabelian gauge theories) has been established for quite some time [4–6]. The idea is to consider noncommutative gauge fields A_μ and gauge transformation parameters Λ that are valued in the enveloping algebra of the gauge group and can be expressed in terms of the ordinary gauge field a_μ, gauge parameter λ and the noncommutative parameter $\theta^{\mu\nu}$ in such a way that an ordinary gauge transformations of a_μ induces a non-commutative gauge transformation of $A_\mu[a]$ with non-commutative gauge parameter $\Lambda[\lambda, a]$.

Comparing to the simpler formulation in which the gauge group is directly deformed by replacing the normal product of group elements with the Moyal-Weyl star product, the Seiberg-Witten map approach removes the restrictions on the gauge group and charge and

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allows the construction of more realistic models. The noncommutative action can be treated as a complicated action written in terms of ordinary fields, which when expanded in the powers of the noncommutativity parameter θ gives the usual commutative action (both free and interacting parts) at zeroth order in θ plus higher order non-commutative corrections. Therefore, such a theory can be considered to be a minimal noncommutative extension of the corresponding commutative model. Following this line, a minimal noncommutative extension of the standard model has been established [7 − 9] and influences to particle physics have been studied up to loop level in low orders of θ [10, 11]. Besides being useful for phenomenology, the θ-expansion was also shown to be improving the renormalizability of the noncommutative gauge theory [12 − 15]. The photon self energy is renormalizable up to any finite order of θ [12] (with the sacrifice of introducing an infinite number of coupling constant from the freedom/ambiguity within the Seiberg-Witten map).

Although the θ-expansion method works nicely in model building, crucial non-perturbative information is lost due to the cut off at finite order of θ. It is long known that in noncommutative field theories [16 − 19], the Moyal-Weyl star product results in a nontrivial phase factor for the Fourier modes when two functions are multiplied together. Such a phase, when it appears in loop calculations, regulates the ultraviolet divergence in the one loop two point function of both noncommutative ϕ^4 and noncommutative quantum electrodynamics (NCQED) but introduces an infrared divergent term of the form $1/(p\theta\theta p)$. As the nontrivial phase factor appears only when all orders of θ in the star product are summed over, this effect does not show up in the noncommutative gauge theories defined by the Seiberg-Witten map approach when it is studied using the θ-expansion method, (thus it is sometimes claimed that such a theory is free of UV/IR mixing). However, as already suggested in some very early papers [20, 2, 6], the θ-expansion is not the only possible way of expressing the Seiberg-Witten map. As the noncommutative gauge field A_μ is a function of both the ordinary field a_μ and the noncommutativity parameter θ^{ij}, one can, instead of expanding A_μ in power of θ, expand it in powers of a_μ. The first several orders of the expansion can be written in a simple form by introducing certain generalized star products [6, 20]. Such an expansion enables us to treat all orders of θ at once in each interaction vertex, thereby allowing us to compute nonperturbative results. In this article we are going to use this expansion to compute the fermion one loop correction to the photon two point function of a NCQED model defined by Seiberg-Witten map. We will see that UV/IR mixing will still arise via the nontrivial phase factors, hence the absence of UV/IR mixing in the Seiberg-Witten map approach to noncommutative gauge theory so far has been really a technical artifact of the perturbative θ-expansion method, but not a feature of the theory itself.

2. The model

For simplicity we consider a NCQED model with a U(1) gauge field A_μ and a fermion field Ψ which lives in the adjoint representation of the noncommutative gauge group $\mathrm{U}(1)_\star$. The action is as follows[1]

$$S=\int -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+i\bar{\Psi}\not{D}\Psi \tag{2.1}$$

with

$$D_\mu\Psi=\partial_\mu\Psi-i[A_\mu \stackrel{\star}{,} \Psi] \quad \text{and} \quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-i[A_\mu \stackrel{\star}{,} A_\nu]$$

The θ-exact Seiberg-Witten map can be obtained in several ways: From the closed formula derived using deformation quantization based on Kontsevich formality map [2], by the relationship between open Wilson lines in the commutative and noncommutative picture [20], or by a direct recursive computation using consistency conditions. The computation of the one loop two-point function requires fully θ-exact interaction vertices up to four external legs, i.e. one needs the θ-exact Seiberg-Witten map of A_μ up to third order in a_μ. This has been computed in its inverse form (a_μ in terms of A_μ up to A^3) in [20]. Here we simply take the inverse of this result by matching the trivial identity $a_\mu(A_\mu(a_\mu))=a_\mu$ order by order, resulting in

$$\begin{aligned}A_\mu\,=\,&a_\mu-\frac{1}{2}\theta^{ij}a_i\star_2(\partial_j a_\mu+f_{j\mu})+\frac{1}{2}\theta^{ij}\theta^{kl}\bigg\{\frac{1}{2}(a_k\star_2(\partial_l a_i+f_{li}))\star_2(\partial_j a_\mu+f_{j\mu})\\&+a_i\star_2(\partial_j(a_k\star_2(\partial_l a_\mu+f_{l\mu}))-\frac{1}{2}\partial_\mu(a_k\star_2(\partial_l a_j+f_{lj})))-a_i\star_2(\partial_k a_j\star_2\partial_l a_\mu)+\\&[a_i\partial_k a_\mu(\partial_j a_l+f_{jl})-\partial_k\partial_i a_\mu a_j a_l+2\partial_k a_i\partial_\mu a_j a_l]_{\star_3}\bigg\}+\mathcal{O}(A^4)\end{aligned} \tag{2.2}$$

where $\star,\star_2,\star_3$ are Moyal-Weyl star product and two generalized star products:

$$f(x)\star g(x)=e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial y^\mu}\frac{\partial}{\partial z^\nu}}f(y)g(z)\bigg|_{x=y=z} \tag{2.3}$$

$$f(x)\star_2 g(x)=\frac{\sin\frac{\partial_1\wedge\partial_2}{2}}{\frac{\partial_1\wedge\partial_2}{2}}f(x_1)g(x_2)\bigg|_{x_1=x_2=x} \tag{2.4}$$

$$[f(x)g(x)h(x)]_{\star_3}\,=\,\Big[\frac{\sin(\frac{\partial_2\wedge\partial_3}{2})\sin(\frac{\partial_1\wedge(\partial_2+\partial_3)}{2})}{\frac{(\partial_1+\partial_2)\wedge\partial_3}{2}\,\frac{\partial_1\wedge(\partial_2+\partial_3)}{2}}+\{1\leftrightarrow 2\}\Big]f(x_1)g(x_2)h(x_3)\bigg|_{x_i=x} \tag{2.5}$$

where

$$\partial_1\wedge\partial_2=\theta^{ij}\frac{\partial}{\partial x_1^i}\frac{\partial}{\partial x_2^j} \tag{2.6}$$

The expansion for a matter particle in the adjoint representation of $\mathrm{U}(1)_\star$ can be easily obtained by taking the linear part (linear operator acting on a_μ) in the expansion of A_μ, which leads to following result:

$$\begin{aligned}\Psi=\psi-\theta^{ij}a_i\star_2\partial_j\psi+\frac{1}{2}\theta^{ij}\theta^{kl}\bigg\{&(a_k\star_2(\partial_l a_i+f_{li}))\star_2\partial_j\psi+2a_i\star_2(\partial_j(a_k\star_2\partial_l\psi))\\&-a_i\star_2(\partial_k a_j\star_2\partial_l\psi)-\big[a_i\partial_k\psi(\partial_j a_l+f_{jl})-\partial_k\partial_i\psi a_j a_l\big]_{\star_3}\bigg\}+\mathcal{O}(a^3)\psi\end{aligned} \tag{2.7}$$

[1] We use a Minkowskian signature here. In the next section we allow a Wick rotation, thus the result is actually on noncommutative R^4. This procedure is the same as taken in [17 – 19], but differs from a procedure where the action is directly written down in R^4.

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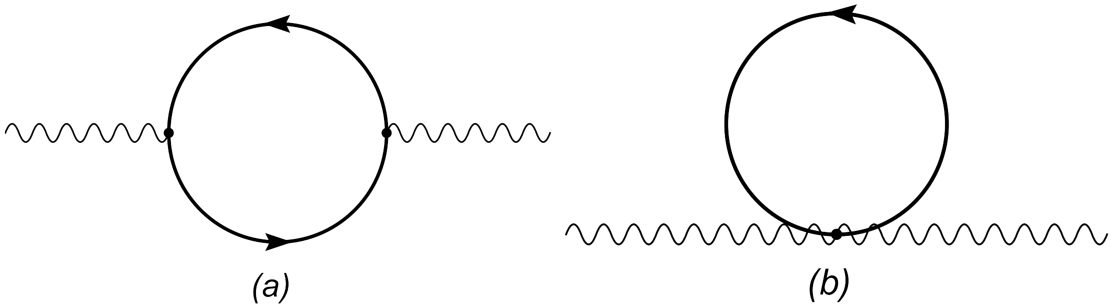

Figure 1: Fermion loop corrections to the photon self energy.

Now the action can be expanded as following:

$$S = \int -\frac{1}{4} f^{\mu\nu} f_{\mu\nu} + i\bar{\psi}\partial\!\!\!/\psi + L_{pp} + L_{pf} \tag{2.8}$$

L_{pf} and L_{pp} are photon-fermion and photon self interaction terms, in this article we concentrate on the photon-fermion part, so we write out L_{pf} explicitly:

$$\begin{aligned} L_{pf} =\ & \bar{\psi}\gamma^\mu[a_\mu \overset{\star}{,} \psi] + i(\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\partial\!\!\!/\psi - i\bar{\psi} \star \partial\!\!\!/(\theta^{ij} a_i \star_2 \partial_j\psi) + (\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\gamma^\mu[a_\mu \overset{\star}{,} \psi] \\ & -\bar{\psi}\gamma^\mu[a_\mu \overset{\star}{,} \theta^{ij} a_i \star_2 \partial_j\psi] - \bar{\psi}\gamma^\mu\left[\frac{1}{2}\theta^{ij} a_i \star_2 (\partial_j a_\mu + f_{j\mu}) \overset{\star}{,} \psi\right] - i(\theta^{ij}\partial_i\bar{\psi}\star_2 a_j)\partial\!\!\!/(\theta^{kl} a_k \star_2 \partial_l\psi) \\ & + \frac{i}{2}\theta^{ij}\theta^{kl}\big((a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j\bar{\psi} + 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l\bar{\psi})) - a_i \star_2 (\partial_k a_j \star_2 \partial_l\bar{\psi}) \\ & + \big[a_i\partial_k\bar{\psi}(\partial_j a_l + f_{jl}) - \partial_k\partial_i\bar{\psi} a_j a_l\big]_{\star 3}\big)\partial\!\!\!/\psi + \frac{i}{2}\theta^{ij}\theta^{kl}\bar{\psi}\partial\!\!\!/\big((a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j\psi \\ & + 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l\psi)) - a_i \star_2 (\partial_k a_j \star_2 \partial_l\psi) + \frac{1}{2}\theta^{ij}\theta^{kl}\big[a_i\partial_k\psi(\partial_j a_l + f_{jl}) - \partial_k\partial_i\psi a_j a_l\big]_{\star 3}\big) \\ & + \bar{\psi}\mathcal{O}(a^3)\psi \end{aligned} \tag{2.9}$$

One noticeable feature of L_{pf} is that it contains vertices identical to NCQED (without Seiberg-Witten map) in leading order instead of ordinary QED as the θ-expanded approach does. This observation holds also for L_{pp}. One thus knows that the one-loop two point function will contain UV/IR mixing terms coming from those integrals in the same way as NCQED. The question is only whether there will be new corrections coming from terms that arise solely due to Seiberg-Witten map or not. As we will see in the next section, for the fermion loop, the leading order IR divergent result is fully identical for NCQED with and without Seiberg-Witten map.

3. One-loop computation

The free part of the action (2.8) is completely identical to ordinary commutative QED, hence the quantization is straightforward. Vertices coming from the fermion-photon interaction lagrangian (2.9) are listed in the appendix. The fermion loop contribution to the one loop photon two point function contains two diagrams: the normal vacuum polarization graph as shown in as shown in figure 1(a) and a new fermion tadpole graph in figure 1(b).

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Diagram (a) leads to following integral:

$$
\begin{aligned}
i\Pi^{ij}_{3-3} =& -4i\int \frac{d^4k}{i(2\pi)^4}\frac{1}{(k+\frac{p}{2})^2(k-\frac{p}{2})^2}\sin^2\frac{p\wedge k}{2}\,\mathrm{tr}\left\{\left[\gamma^i\left(\not{k}+\frac{\not{p}}{2}\right)\gamma^j\left(\not{k}-\frac{\not{p}}{2}\right)\right]\right.\\
&+\frac{1}{p\wedge k}\left[(\tilde{p}^i\not{k}-\tilde{k}^i\not{p})\left(\not{k}+\frac{\not{p}}{2}\right)\gamma^j\left(\not{k}-\frac{\not{p}}{2}\right)+\gamma^i\left(\not{k}+\frac{\not{p}}{2}\right)(\tilde{p}^j\not{k}-\tilde{k}^j\not{p})\left(\not{k}-\frac{\not{p}}{2}\right)\right]\\
&\left.+\frac{1}{(p\wedge k)^2}\left[(\tilde{p}^i\not{k}-\tilde{k}^i\not{p})\left(\not{k}+\frac{\not{p}}{2}\right)(\tilde{p}^j\not{k}-\tilde{k}^j\not{p})\left(\not{k}-\frac{\not{p}}{2}\right)\right]\right\}
\end{aligned}
\tag{3.1}
$$

where $\tilde{p}^i = \theta^{ij}p_j$.

For diagram (b) we get a surprising result: Its contribution can be shown to vanish:

$$
\begin{aligned}
i\Pi^{ij}_4 =& \; i\int \frac{d^4k}{i(2\pi)^4}\frac{1}{k^2}\,\mathrm{tr}\left\{4\frac{\sin^2\frac{p\wedge k}{2}}{p\wedge k}(\tilde{k}^{i_1}\not{k}\gamma^{i_2}+\tilde{k}^{i_2}\not{k}\gamma^{i_1})-4\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}(\not{k}\not{p}+\not{k}\not{k})\tilde{k}^{i_1}\tilde{k}^{i_2}\right.\\
&+2\not{k}\not{k}\left[(-2\tilde{p}^{i_1}\tilde{k}^{i_2}+p\wedge k\theta^{i_1i_2})+\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}2(\tilde{k}-\tilde{p})^{i_1}\tilde{k}^{i_2}+\frac{\sin^2\frac{p\wedge k}{2}}{p\wedge k}\theta^{i_1i_2}+\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}(2\tilde{k}^{i_2}\tilde{p}^{i_1}\right.\\
&+\theta^{i_1i_2}k\wedge p)-\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}\tilde{k}^{i_1}\tilde{k}^{i_2}+(2\tilde{p}^{i_1}\tilde{k}^{i_2}-p\wedge k\theta^{i_1i_2})+\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}2(\tilde{k}+\tilde{p})^{i_1}\tilde{k}^{i_2}\\
&\left.-\frac{\sin^2\frac{p\wedge k}{2}}{p\wedge k}\theta^{i_1i_2}-\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}(2\tilde{k}^{i_2}\tilde{p}^{i_1}+\theta^{i_1i_2}k\wedge p)-\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}\tilde{k}^{i_1}\tilde{k}^{i_2}\right]-4\frac{\sin^2\frac{p\wedge k}{2}}{p\wedge k}(\tilde{k}^{i_1}\not{k}\gamma^{i_2}\\
&+\tilde{k}^{i_2}\not{k}\gamma^{i_1})+4\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}(\not{k}\not{p}-\not{k}\not{k})\tilde{k}^{i_1}\tilde{k}^{i_2}+2\not{k}\not{k}\left[(2\tilde{p}^{i_2}\tilde{k}^{i_1}-p\wedge k\theta^{i_2i_1})+\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}2(\tilde{k}+\tilde{p})^{i_2}\tilde{k}^{i_1}\right.\\
&-\frac{\sin^2\frac{p\wedge k}{2}}{p\wedge k}\theta^{i_2i_1}-\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}(2\tilde{k}^{i_1}\tilde{p}^{i_2}+\theta^{i_2i_1}k\wedge p)-\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}\tilde{k}^{i_1}\tilde{k}^{i_2}-(2\tilde{p}^{i_1}\tilde{k}^{i_2}-p\wedge k\theta^{i_1i_2})\\
&\left.\left.+\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}2(\tilde{k}-\tilde{p})^{i_2}\tilde{k}^{i_1}+\frac{\sin^2\frac{p\wedge k}{2}}{p\wedge k}\theta^{i_2i_1}-\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}(2\tilde{k}^{i_2}\tilde{p}^{i_1}+\theta^{i_1i_2}k\wedge p)+\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}\tilde{k}^{i_1}\tilde{k}^{i_2}\right]\right\}\\
=& \; i\int\frac{d^4k}{i(2\pi)^4}\frac{1}{k^2}tr\left\{-8\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}\not{k}\not{k}\tilde{k}^{i_1}\tilde{k}^{i_2}+8\frac{\sin^2\frac{p\wedge k}{2}}{(p\wedge k)^2}\not{k}\not{k}\tilde{k}^{i_1}\tilde{k}^{i_2}\right\}\\
=& \; 0
\end{aligned}
\tag{3.2}
$$

Hence we only need to evaluate the integral (3.1). We work out the trace in (3.1), then write the wedge product in its explicit component form, to obtain:

$$
\begin{aligned}
i\Pi^{ij} =& -16i\int\frac{d^4k}{i(2\pi)^4}\frac{1}{(k+\frac{p}{2})^2(k-\frac{p}{2})^2}\sin^2\frac{p_i\theta^{ij}k_j}{2}\left\{\left[2k^ik^j-k^2g^{ij}-\frac{1}{4}(2p^ip^j-p^2g^{ij})\right]\right.\\
&-\frac{1}{p_i\theta^{ij}k_j}\left[2(p\cdot k)(\tilde{k}^ik^j+k^i\tilde{k}^j)-\left(k^2+\frac{p^2}{4}\right)(\tilde{p}^ik^j+k^i\tilde{p}^j+\tilde{k}^ip^j+p^i\tilde{k}^j)+\frac{1}{2}(p\cdot k)(\tilde{p}^ip^j\right.\\
&\left.+p^i\tilde{p}^j)\right]+\frac{1}{(p_i\theta^{ij}k_j)^2}\left[\left(k^4-\frac{(p\cdot k)^2}{2}+\frac{p^2k^2}{4}\right)\tilde{p}^i\tilde{p}^j-\left(p^2k^2-2(p\cdot k)^2+\frac{p^4}{4}\right)\tilde{k}^i\tilde{k}^j\right.\\
&\left.\left.-\left(k^2-\frac{p^2}{4}\right)(p\cdot k)(\tilde{p}^i\tilde{k}^j+\tilde{k}^i\tilde{p}^j)\right]\right\}
\end{aligned}
\tag{3.3}
$$

As expected, we have here in the first square bracket terms that are identical to ordinary NCQED. In the next pair of square brackets are the new contribution coming from the

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Seiberg-Witten map together with the non-trivial IR-divergent coefficients $1/(p_i\theta^{ij}k_j)^n$, where n equals to one for the second and two for the third term. The integral in seems to be not very different to its counterpart in normal NCQED. Previous results [18, 19] suggest that one can rewrite

$$\sin^2 \frac{p_i\theta^{ij}k_j}{2} = \frac{1}{2}(1-\cos(p_i\theta^{ij}k_j)) \tag{3.4}$$

to separate terms with and without nontrivial phase shift (planar and non-planar). However, the IR-divergent term $1/(p_i\theta^{ij}k_j)^n$ introduces unexpected difficulties to the usual renormalization procedure. The term $1/(p_i\theta^{ij}k_j)$ cannot be removed by introducing a Schwinger parameter as it does not have a fixed sign in R^4. Furthermore, the term $1/(p_i\theta^{ij}k_j)^2$ leads to a complicated Gaussian integral over k_μ whose convergence in R^4 depends on the explicit choice of p_μ (instead of p^2). Here, we try to evaluate the leading order non-planar part by the following trick: We introduce an additional variable λ in the sine function in (3.3) to make it $\sin\lambda(p_i\theta^{ij}k_j)$, then one can cancel the negative power of $(p_i\theta^{ij}k_j)$ by taking an appropriate number of derivatives w.r.t. λ. The resulting integral is:

$$\begin{aligned}
i\Pi''^{ij}(\lambda) = & -8i\int \frac{d^4k}{i(2\pi)^4}\frac{\cos(\lambda p_i\theta^{ij}k_j)}{(k+\frac{p}{2})^2(k-\frac{p}{2})^2}\Bigg\{(p_i\theta^{ij}k_j)^2\left[2k^ik^j-k^2g^{ij}-\frac{1}{4}(2p^ip^j-p^2g^{ij})\right] \\
& -(p_i\theta^{ij}k_j)\left[2(p\cdot k)(\tilde{k}^ik^j+k^i\tilde{k}^j)-\left(k^2+\frac{p^2}{4}\right)(\tilde{p}^ik^j+k^i\tilde{p}^j+\tilde{k}^ip^j+p^i\tilde{k}^j)\right. \\
& \left.+\frac{1}{2}(p\cdot k)(\tilde{p}^ip^j+p^i\tilde{p}^j)\right]+\left[\left(k^4-\frac{(p\cdot k)^2}{2}+\frac{p^2k^2}{4}\right)\tilde{p}^i\tilde{p}^j-\left(p^2k^2-2(p\cdot k)^2+\frac{p^4}{4}\right)\tilde{k}^i\tilde{k}^j\right. \\
& \left.-\left(k^2-\frac{p^2}{4}\right)(p\cdot k)(\tilde{p}^i\tilde{k}^j+\tilde{k}^i\tilde{p}^j)\right]\Bigg\}
\end{aligned} \tag{3.5}$$

The rest of the computation follows the standard dimensional regulation method. By taking derivatives w.r.t. λ the integral (3.5) is more divergent than (3.5) without the additional phase factor (the cosine function in our case), fortunately we know the effective UV regulator coming from the cosine decays exponentially, therefore is still effective in this case. Now one integrates over λ and evaluates the resulting function at $\lambda = 1$. The free integration constant can be fixed by matching the result for the first square bracket to the direct computation in NCQED. Finally we obtained the following (surprisingly simple) result for the leading order IR divergent term:

$$\Pi^{ij}_{non-planar} = -\frac{64}{16\pi^2}\frac{\tilde{p}^i\tilde{p}^j}{\tilde{p}^4} \tag{3.6}$$

which is identical to the corresponding result in normal NCQED.

4. Conclusion

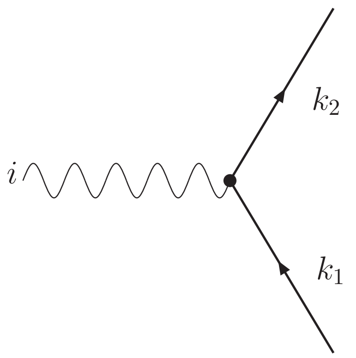
By explicit computation we have shown that NCQED defined via Seiberg-Witten map still exhibits UV/IR mixing in its photon one-loop two-point function, when this theory is treated nonperturbatively in θ. The proof of principle that this nonperturbative computation can be done at all is perhaps the most important result of this work. To find the full expression for the UV/IR mixing term one needs to compute also the photon self interaction loop corrections, which can be done by a procedure practically identical to the computation of the fermion loop. By the arguments given in section 2, we know that there exists in general also UV/IR mixing terms in the photon loop correction. Hence it is quite safe to say that UV/IR mixing still exists in noncommutative quantum gauge theories constructed using Seiberg-Witten maps and one still needs to worry about unusual large modifications to the very low energy physics from arbitrarily small θ since the $\theta \to 0$ limit is discontinuous at the quantum level.

Besides UV/IR mixing, the θ-exact approach gives rise to a regularization problem, which requires some improvement in the renormalization procedure. From this view point the θ-expansion method in [12] seems to be more convenient. Another possible candidate is the Hamiltonian approach to renormalization, which has successfully achieved finite results for noncommutative scalar field theory in Minkowski space-time [21]. Another approach [22] to NCQED based on the Yang-Feldman equation encountered similar problems for the photon two point function as we encountered here.[2]

Acknowledgments

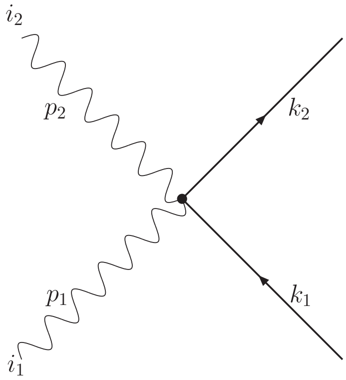
Helpful discussions with Robert C. Helling are gratefully acknowledged.

A. Feynman rules for photon-fermion interaction



$$V^i_{pff}(k_1, k_2) = 2\gamma^i \sin\frac{k_1 \wedge k_2}{2} + 2(\tilde{k}_1^i \not{k}_2 - \tilde{k}_2^i \not{k}_1)\frac{\sin\frac{k_1\wedge k_2}{2}}{k_1 \wedge k_2} \tag{A.1}$$

[2]It is worth also to mention that in [22] an inexplicit expansion of open-Wilson lines is constructed up to arbitrary formal order of the gauge field, while the author probably did not notice the connection between the expansion of open-Wilson lines and Seiberg-Witten maps and erroneously claims that the Seiberg-Witten map is only valid in an θ-expanded way.



$$
\begin{aligned}
V^{i_1 i_2}_{ppff}(p_1,p_2,k_1,k_2) = \Bigg\{ & 4i\frac{\sin\frac{p_1\wedge k_1}{2}\sin\frac{p_2\wedge k_2}{2}}{p_1\wedge k_1}\tilde{k}_1^{i_1}\gamma^{i_2} - 4i\frac{\sin\frac{p_1\wedge k_1}{2}\sin\frac{p_2\wedge k_2}{2}}{p_2\wedge k_2}\tilde{k}_2^{i_2}\gamma^{i_1} \\
& - 2i\frac{\sin\frac{k_1\wedge k_2}{2}\sin\frac{p_1\wedge p_2}{2}}{p_1\wedge p_2}(2\gamma^{i_2}\tilde{p}_2^{i_1} - \not{p}_2\theta^{i_1 i_2}) - 4i\frac{\sin\frac{p_1\wedge k_1}{2}\sin\frac{p_2\wedge k_2}{2}}{p_1\wedge k_1 p_2\wedge k_2}(\not{p}_2 \\
& + \not{k}_2)\tilde{k}_1^{i_1}\tilde{k}_2^{i_2} + 2i\not{k}_2\Bigg[\frac{\sin\frac{k_1\wedge k_2}{2}\sin\frac{p_1\wedge p_2}{2}}{p_1\wedge p_2 k_1\wedge k_2}(p_2\wedge k_1\theta^{i_1 i_2} - 2\tilde{p}_2^{i_1}\tilde{k}_1^{i_2}) \\
& - \frac{\sin\frac{p_1\wedge k_2}{2}\sin\frac{p_2\wedge k_1}{2}}{p_1\wedge k_2 p_2\wedge k_1}2(\tilde{p}_2-\tilde{k}_1)^{i_1}\tilde{k}_1^{i_2} + \frac{\sin\frac{p_1\wedge k_2}{2}\sin\frac{p_2\wedge k_1}{2}}{p_1\wedge k_2}\theta^{i_1 i_2} \\
& + \left(\frac{\sin\frac{p_2\wedge k_1}{2}\sin\frac{p_1\wedge k_2}{2}}{p_2\wedge k_2 p_1\wedge k_2} + \frac{\sin\frac{p_1\wedge p_2}{2}\sin\frac{k_1\wedge k_2}{2}}{p_2\wedge k_2 k_1\wedge k_2}\right)(2\tilde{k}_1^{i_2}\tilde{p}_2^{i_1} + \theta^{i_1 i_2}k_1\wedge p_2 \\
& - \tilde{k}_1^{i_1}\tilde{k}_1^{i_2})\Bigg] + 2i\not{k}_1\Bigg[\frac{\sin\frac{k_2\wedge k_1}{2}\sin\frac{p_1\wedge p_2}{2}}{p_1\wedge p_2 k_2\wedge k_1}(2\tilde{p}_2^{i_1}\tilde{k}_2^{i_2} - p_2\wedge k_2\theta^{i_1 i_2}) \\
& + \frac{\sin\frac{p_1\wedge k_1}{2}\sin\frac{p_2\wedge k_2}{2}}{p_1\wedge k_1 p_2\wedge k_2}2(\tilde{p}_2+\tilde{k}_2)^{i_1}\tilde{k}_2^{i_2} - \frac{\sin\frac{p_1\wedge k_1}{2}\sin\frac{p_2\wedge k_2}{2}}{p_1\wedge k_1}\theta^{i_1 i_2} \\
& - \left(\frac{\sin\frac{p_2\wedge k_2}{2}\sin\frac{p_1\wedge k_1}{2}}{p_2\wedge k_1 p_1\wedge k_1} + \frac{\sin\frac{p_2\wedge p_1}{2}\sin\frac{k_2\wedge k_1}{2}}{p_2\wedge k_1 k_2\wedge k_1}\right)(2\tilde{k}_2^{i_2}\tilde{p}_2^{i_1} + \theta^{i_1 i_2}k_2\wedge p_2 \\
& + \tilde{k}_2^{i_1}\tilde{k}_2^{i_2})\Bigg] + \{p_1\leftrightarrow p_2 \,\text{and}\, i_1\leftrightarrow i_2\}\Bigg\}\delta(k_1-k_2-p_1-p_2)
\end{aligned}
\tag{A.2}
$$

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